Name: \_\_SOLUTIONS\_\_\_\_\_\_\_\_

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Full Test (Calc Free + Calc Assumed)

Total Time: 45 minutes

Total Marks: 38 marks

Student Result \_\_\_\_\_\_\_\_/ 38

**MATHEMATICS METHODS Unit 3**

**TEST 1 -2024: Further differentiation, integration and applications.**

**Part A**

**Calculator Free Section**

Time: 25 minutes

Total Marks: \_\_\_\_\_\_ / 21 marks

Resources allowed: SCSA Formula Sheet

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks**. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 1** | **[2, 4 = 6 marks]** |

A body moves under rectilinear motion such that for time seconds, , the body’s displacement, metres, from the origin is given by:

a) Determine its acceleration at .

✓uses first derivative to determine velocity

✓uses second derivative to determine acceleration

and evaluates at

b) Use calculus techniques and the second derivation to determine and justify when the displacement is

at a maximum.

Max displacement and

and

✓recognizes that max displacement occurs when and

or ✓solves

and ✓uses second derivative, , to test nature of stationary points (concavity)

Therefore, concave down at (local maximum)

Max displacement when ✓states maximum displacement occurs when

|  |  |
| --- | --- |
| **Question 2** | **[3, 2 = 5 marks]** |

Determine for each of the following. Do not simplify your answer.

a)

✓determines

✓determines

✓uses quotient rule correctly and divides by

b)

✓uses chain rule (may use substitution and )

✓correctly differentiates

|  |  |
| --- | --- |
| **Question 3** | **[4, 2 = 6 marks]** |

a) For the function, , determine the coordinates of any points of inflection and the gradient of the curve at these points.

✓determines 1st and 2nd derivatives

or ✓solves 2nd derivative = 0

Inflection points at and ✓determines coordinates of inflection points

gradient at

gradient at

✓determines gradient of curve at inflection points

b) One of the stationary points on the curve is located at the point . Use a calculus method to determine the nature of this stationary point.

✓uses the 2nd derivative to check concavity

positive 2nd derivative, therefore, concave up when

stationary point is a local minimum. ✓concludes a local minimum

|  |  |
| --- | --- |
| **Question 4** | **[2, 2 = 4 marks]** |

Determine each of the following indefinite integrals.

a)

✓correct numerator (function)

✓correct denominator (coefficient) and

b)

✓correct numerator (function)

✓correct denominator of 1 (coefficient) and

Name: \_\_SOLUTIONS\_\_\_\_\_\_\_\_

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**MATHEMATICS METHODS Unit 3**

**TEST 1 -2024: Further differentiation, integration and applications.**

**Part A**

**Calculator Assumed Section**

Time: 20 minutes

Total Marks: \_\_\_\_\_\_ / 17 marks

Resources allowed:

SCSA Formula Sheet

Up to three Calculators and

One A4 sheet, both sides of notes

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. **For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 5** | **[4, 2, 2, 1 = 9 marks]** |

A sphere of radius 30 cm is sitting on the base of a container of water. The water level is rising so that the sphere is gradually becoming submerged.

Let be the height of the water, in centimetres, as illustrated below left. The submerged portion of the sphere is as shown below right.

Diagram, venn diagram

Description automatically generated A picture containing diagram

Description automatically generated

The volume of the submerged portion, where is the radius of the sphere, is given by

a) Using the increments formula, calculate to 1 decimal place the approximate increase in submerged volume for a cm increase in height when cm.

Since cm

✓determines and substitutes cm

cm ✓states

✓correctly applies the incremental change formula

cm3 ✓correctly evaluates approximate change in volume

Submerged volume increases by approx. cm3

Consider that the water level was rising at a constant rate of mm per second, so that the height of the water in the container since initially starting dry, was given by .

b) Using the chain rule, show that , the rate of change in the submerged volume of the sphere with respect to time.

✓indicates correct use of chain rule to obtain

from part a)

✓uses chain rule and writes in terms of

c) Determine the time taken for the rate of change in the submerged volume of the sphere to reach a maximum.

✓indicates is required

Solving

seconds ✓correct time

d) At what height will the water be in the container for the rate of change in the submerged volume of the sphere be at a maximum? How does this relate to the size of the sphere?

cm

Height of the water in the container will be 30 cm.

This means the water has the sphere half submerged.

✓states water will have the sphere half submerged or similar

|  |  |
| --- | --- |
| **Question 6** | **[2, 1, 4, 1 = 8 marks]** |

<EFOFEX>
id:fxd{cf27c148-e005-4e69-a59a-6a8d52f0dae4}

FXData:
</EFOFEX>A rectangular beam is to be cut from a non-circular tree trunk whose cross-sectional outline can be represented by the equation .

a) Show that the area of the cross-section of the beam is given by where is the half-width of the beam.

✓indicates

positive only for a positive area

✓substitutes and simplifies

b) State the possible values for .

Since

, however, if is the half-width and area requires to be positive,

then ✓correct value range

c) Find the value of for which the cross-sectional area of the beam is a maximum and find the corresponding value of .

✓uses product rule

✓correct derivative

Note: derivative can be shown (not required) to simplify to:

Solve using CAS

✓solves to obtain value for

✓obtains value for

d) Find the maximum cross-sectional area of the beam.

units2

units2 ✓correct maximum area (follow through for c))

**End of Test**